Discovery of Lagrange multipliers and Lagrange mechanics

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The talk, essentially the first part of the article [1], explains how

- A work of Archimedes (-287 to -212, born 2300 years ago),
- A thick book on statics (Varignon 1725),
- a letter by Johann Bernoulli to Varignon (1715, 300 years ago),
- Euler's *Methodus* (1744, on variational calculus),
- Lagrange's *opus 1 and opus 3* (1759, on "Gaussian Elimination"),
- and d'Alembert's *Dynamique* from 1743,

led to the famous *Mécanique analytique* (1788, 1811) by Lagrange, in which, in the first part, the advantage of the *methods of multipliers* is demonstrated at many examples and, in the second part, the *equations of Lagrange dynamics* are derived from the principle of least action.

In the last part of the talk we show the connection of the ideas of Euler and Lagrange with problems of optimal control (Carathéodory, Pontryagin).

References

[1] M.J. Gander, F. Kwok and G. Wanner, Constrained Optimization: from Lagrangian Mechanics to Optimal Control and PDE Constraints , Optimization with PDE Constraints, LNCS 101, pp. 151–202, 2014.