Classification of von Neumann algebras and their quantum symmetries

Stefaan Vaes (*KU Leuven*)

In this talk, I present an overview of recent progress in the structural theory of von Neumann algebras and their symmetries. Von Neumann algebras arise in many different ways, most notably as the group von Neumann algebra $L(\Gamma)$ of a countable group Γ and as the group measure space construction $L^\infty(X) \rtimes \Gamma$ associated to an action of a countable group Γ on a probability space (X, μ).

Using Sorin Popa's deformation/rigidity theory, several classification theorems for von Neumann algebras were obtained in recent years. In the case of crossed products $M = L^\infty(X) \rtimes \Gamma$, a crucial role is played by the canonical subalgebra $A = L^\infty(X)$. When the action of Γ on (X, μ) is free, $A \subset M$ is a Cartan subalgebra. I will discuss the question of existence and uniqueness of Cartan subalgebras, as well as other "regular" subalgebras, and its primary importance in the classification of group measure space von Neumann algebras, [1], and other families of crossed products, [3]. I will also focus on remarkable families of von Neumann algebras having several Cartan subalgebras.

Von Neumann algebras M have a very symmetric structure, described by the automorphisms of M, but also by their "quantum symmetries" given as M-bimodules of finite Jones index. These quantum symmetries especially appear in Jones' theory of subfactors. In this way, a finite index subfactor $N \subset M$ can be encoded as an action on M by a discrete group like structure. These discrete group like structures have been axiomatized as λ-lattices by Popa and as planar algebras by Jones. I will discuss the recent geometric group theoretic approach to λ-lattices and planar algebras, [2], and also emphasize some of the important open problems in this direction, on the existence and uniqueness of actions of λ-lattices on the hyperfinite II_1 factor.

References

[1] S. Popa, S. Vaes. Unique Cartan decomposition for II_1 factors arising from arbitrary actions of free groups. Acta Math. 212 (2014), 141–198.

[2] S. Popa, S. Vaes. Representation theory for subfactors, λ-lattices and C*-tensor categories. Comm. Math. Phys. 340 (2015), 1239–1280.

[3] S. Vaes, P. Verraedt. Classification of type III Bernoulli crossed products. Adv. Math. 281 (2015), 296–332.