DeGiorgi-Nash-Moser and Hörmander theories: new interplays

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We report on recent results at the crossroad of two major theories in the analysis of partial differential equations. The celebrated DeGiorgi-Nash-Moser theorem provides Hölder estimates and the Harnack inequality for uniformly elliptic or parabolic equations with rough coefficients in divergence form. The theory of hypoellipticity of Hörmander provides general “bracket” conditions for regularity of solutions to partial differential equations combining first and second order derivative operators when ellipticity fails in some directions.

We extend the DeGiorgi-Nash-Moser theory to a class of equations sharing the same formal structure as the hypoelliptic equations "of type II”, sometimes also called ultra-parabolic equations of Kolmogorov type, but with rough coefficients. More precisely, these equations combine a first-order skew-symmetric operator with a second-order elliptic operator involving derivatives along only part of the coordinates and with rough coefficients. The so-called averaging lemmas play a key role in the proof.

We obtain as a corollary new results for nonlinear Vlasov-Fokker-Planck and Landau equations in kinetic theory. We will discuss the new questions and perspectives opened. This is a joint work with François Golse, Cyril Imbert and Alexis Vasseur.