

Phase transitions in discrete structures

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A wide variety of problems in combinatorics, computer science, information theory and mathematical physics can be described along the following lines. There are a large number of ‘variables’, each ranging over a small finite domain, and a number of ‘constraints’ of a similar order of magnitude. Each constraint binds a few of the variables and encourages or discourages certain value combinations. Examples include the graph coloring problem, the k -SAT problem or the Ising model. In each case, the interaction between the variables and the preferences of the constraints give rise to a probability measure on the possible assignments of values to variables, the *Gibbs measure*.

Furthermore, in many important applications (such as low-density parity check codes) the interactions between the constraints and the variables are random themselves, rendering the Gibbs measure a random object. The fundamental question is: what properties does a typical Gibbs measure enjoy in the limit as the number of variables/constraints tend to infinity?

Over the past 20 years physicists have developed an ingenious but non-rigorous technique for answering this question called the *cavity method* [1]. It applies almost mechanically and yields intriguing “predictions” on long-standing problems. The obvious question that arises is to what extent the cavity method can be put on a rigorous foundation. The talk deals with the recent rigorous progress on this question.

References

- [1] M. Mezard, A. Montanari: Information, physics and computation. Oxford University Press 2008.