Banach spaces with the "calar-plus-compact" and "invariant subspace" properties.

Spiros Argyros (NTUA)

We will discuss two structural properties of the space of boundedlinear operators $\mathcal{L}(X)$ of a Banach space X. The firstone is the "scalar-plus-compact" property of X, concerningBanach spaces X such that every $T \in \mathcal{L}(X)$ is of theform $\lambda I + K$ with I the identity operator and K acompact one. The first space with the "scalar-plus-compact"property was presented in [2] and more results were given in[1], [5]. A common feature of all thoseexamples is the Bourgain-Delbaen $\mathcal{L}^{\infty}$-structure[7]. In this talk we will explain how the later affects thestructure of $\mathcal{L}(X)$.The second property of $\mathcal{L}(X)$ concerns the invariantsubspace one. Namely, whether every $T \in \mathcal{L}(X)$ admits anon trivial closed invariant subspace. By classical results([6]), every space X with the "scalar-plus-compact"property, also satisfies the invariant subspace property. Inparticular, the space $\mathfrak{X}_K$ presented in [2] isthe first known example with that property. On the other hand, in[8], [9], [10], examples of bounded operators onnon reflexive spaces admitting no closed invariant subspaces areexhibited. In the second part of the talk we will present thefirst known examples of reflexive spaces with the invariantsubspace property ([3], [4]).

References

[1] S.A. Argyros, D. Freeman, R. Haydon, E. Odell, Th. Raikoftsalis, Th. Schlumprecht,and D. Zisimopoulou, *embedding uniformly convex spaces intospaces with very few operators*, J. Funct. Anal. **262**, No.3 (2012), 825-849.

[2] S. A. Argyros and R. G. Haydon, *A hereditarilyindecomposable $\mathcal{L}_{\infty}$-space that solves thescalar-plus-compact problem*, Acta Math. **206**, no. 1 (2011),1–54.

[3] S. A. Argyros and P. Motakis, *A reflexive hereditarilyindecomposable space with the hereditary invariant subspaceproperty*, Proc. London Math. Soc. **108**, no. 6 (2014),1381-1416.

[4] S. A. Argyros and P. Motakis, *A dual method of constructinghereditarily indecomposable Banach spaces* arXiv:1504.01564

[5] S. A. Argyros and P. Motakis, *The scalar-plus-compactproperty in spaces without reflexive subspaces*, to appear.

[6] M. Aronszajn and K. T. Smith, *Invariant subspaces of completely continuous operators*, Ann. of Math. **60** (1954), 345-350.

[7] J. Bourgain and F. Delbaen, *A class of special$\mathcal{L}_\infty$ spaces*, Acta Math. **145** (1980),155-176.

[8] P. Enflo, *On the invariant subspace problem for Banachspaces*, Acta Math. **158**, no. 3-4 (1987), 213-313.

[9] C. J. Read, *A solution to the invariant subspace problem*,Bull. London Math. Soc. **16**, no. 4 (1984), 337-401.

[10] C. J. Read, *A solution to the invariant subspace problem onthe space* ℓ_1, Bull. London Math. Soc. **17**, no. 4(1985), 305-317.