Around the Möbius function

Kaisa Matomäki (University of Turku), Maksym Radziwill (Rutgers University)

The Möbius function plays a central role in number theory; both the prime number theorem and the Riemann Hypothesis are naturally formulated in terms of the amount of cancellation one gets when summing the Möbius function. In a recent joint work [1] with Maksym Radziwiłł we have shown that the sum of the Möbius function exhibits cancellation in "almost all intervals" of arbitrarily slowly increasing length, i.e. we have shown that, for any $h = h(x) \to \infty$ with $x \to \infty$, one has

$$\sum_{x \leq n \leq x+h} \mu(n) = o(h)$$

for almost all $x \leq X$. This goes beyond what was previously known conditionally on the Riemann Hypothesis.

Our result holds in fact in much greater generality, and has several further applications, some of which I will discuss in the talk. For instance the general result implies that, for any $\varepsilon > 0$, there exists $C = C(\varepsilon) > 0$ such that, for all large enough x, the interval $[x, x + C\sqrt{x}]$ contains numbers whose all prime factors are at most x^ε. This settles a conjecture on "smooth" numbers and is related to the running time of Lenstra's factoring algorithm.

References

[1] K. Matomäki and M. Radziwiłł. Multiplicative functions in short intervals. _Preprint_, available at `http://arxiv.org` as arXiv:1501.04585 [math.NT].