Near-critical scaling limits in 2d statistical physics

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The general problem addressed in this talk will be the following one: how does a system look on large scales when its temperature is tuned to be very near (but not equal to) its critical temperature. We will focus mainly in this talk on the case of 2D percolation. In a joint work started long ago together with Oded Schramm and completed only recently [1, 2], we construct a continuum object which describes the near-critical limit of 2D percolation. This object is called the *near-critical ensemble* and is denoted by $\lambda \in \mathbb{R} \mapsto \omega_\infty(\lambda)$. The case $\lambda \equiv 0$ corresponds to the critical case which is known to be conformally invariant by Smirnov's theorem and which can be described by the now celebrated Schramm-Loewner-Evolution process SLE_κ with $\kappa = 6$. Once λ becomes positive, the process $\omega_\infty(\lambda)$ describes the scaling limit of the super-critical regime. Conformal invariance is broken and only massive versions of SLE_6 remain.

We give in [3] a striking application of this work to the analysis of the *minimal spanning tree*, a celebrated model in combinatorics. We prove in [3] that the minimal spanning tree defined on a triangular lattice $\delta\mathbb{T}$ with mesh δ has a scaling limit (under a suitable topology) as $\delta \searrow 0$. The limiting tree is derived from the above process $\lambda \mapsto \omega_\infty(\lambda)$.

We also discuss unexpected phenomena in the context of near-critical Ising model [4].

References

[1] Christophe Garban, Gábor Pete, Oded Schramm. Pivotal, cluster, and interface measures for critical planar percolation. J. Amer. Math. Soc. 26 (2013), no. 4, 939–1024.

[2] Christophe Garban, Gábor Pete, Oded Schramm. The scaling limits of near-critical and dynamical percolation. *Preprint.* arXiv:1305.5526

[3] Christophe Garban, Gábor Pete, Oded Schramm. The scaling limits of the Minimal Spanning Tree and Invasion Percolation in the plane. *Preprint.* arXiv:1309.0269

[4] Hugo Duminil-Copin, Christophe Garban, Gábor Pete. The near-critical planar FK-Ising model. Comm. Math. Phys. 326 (2014), no. 1, 1–35.