

Spectral synthesis in Hilbert spaces of entire functions

Anton Baranov (Saint Petersburg State University)

Spectral synthesis is the possibility of the reconstruction of any invariant subspace of a linear operator from generalized eigenvectors that it contains. Another version of the spectral synthesis problem is the reconstruction of a vector in a Banach space from its Fourier series with respect to some complete and minimal system. These problems (which go back to J. Wermer and H. Hamburger) were studied in the 1970s by N. Nikolski and A. Markus who constructed examples of compact operators with complete sets of eigenvectors for which the synthesis failed.

It was a long-standing problem in the nonharmonic Fourier analysis whether any complete and minimal system of exponentials in $L^2(-\pi, \pi)$ has the spectral synthesis property. Namely, given a complete and minimal system of exponentials $\{e^{i\lambda t}\}_{\lambda \in \Lambda}$ with the biorthogonal system $\{g_\lambda\}$, is it true that any function $f \in L^2(-\pi, \pi)$ belongs to the closed linear span of its 'harmonics' $(f, g_\lambda)e^{i\lambda t}$? Recently, we answered this question in the negative. At the same time it was shown that the spectral synthesis for exponential systems always holds up to one-dimensional defect.

We also discuss the spectral synthesis problem for systems of reproducing kernels in some Hilbert spaces of entire functions, including de Branges and Fock type spaces (exponential systems correspond to the classical Paley–Wiener space). In the de Branges space setting the problem can be related to the spectral theory of rank one perturbations of compact selfadjoint operators, and we are able to produce unexpected examples of rank one perturbations which do not admit spectral synthesis. The talk is based on joint works with Yurii Belov, Alexander Borichev, and Dmitry Yakubovich.