Circle actions in symplectic geometry

Leonor Godinho (Instituto Superior Técnico - Universidade de Lisboa), Silvia Sabatini (Universität zu Köln)

The problem of determining whether a manifold admits symmetries has been widely studied in mathematics and physics: it is in general hard to determine whether, given a Lie group G and a manifold M, there exists an action of G on M that preserves a prescribed structure. When M is symplectic, for instance when M is the phase space of a particle, having one conserved quantity whose associated (Hamiltonian) flow on the manifold is periodic, is equivalent to having a Hamiltonian circle action.

The following questions are therefore natural: which symplectic manifolds admit symplectic circle actions? What are their topological properties? We will discuss these problems and related results in the case in which the fixed point set is discrete. Although the group structure of the circle is extremely simple, we will see that the existence of such actions imposes many restrictions on the Chern classes and on the equivariant cohomology ring of the manifold. Our tools can be applied to give a proof of the symplectic Petrie conjecture in low dimensions, which concerns the possible Hamiltonian circle actions with minimal fixed point set [1]. Moreover, such tools can be extended to the broader category of almost complex manifolds to obtain lower bounds and divisibility results for the number of fixed points [2]. This lower bound problem is related to the Kosniowski conjecture, which has been open since 1979.

References

[1] L. Godinho, S. Sabatini. New tools for classifying Hamiltonian circle actions with isolated fixed points. Foundations of Computational Mathematics. 14 (2014), 791–860.

[2] L. Godinho, Á. Pelayo, S. Sabatini. Fermat and the number of fixed points of periodic flows. Preprint. arXiv:1404.4541 [math.AT].