# 2D Ising model: correlations via boundary value problems

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In this talk we give a survey of convergence results obtained during the last several years for correlations in the critical 2D Ising model. In particular, it includes the convergence (as the mesh size tends to zero) of properly rescaled spin expectations to the conformal covariant limits predicted by Conformal Field Theory, for arbitrary planar domains.

We start with reviewing the combinatorics of the planar Ising model considered on a general planar graph and the existence of discrete holomorphic observables in the critical model on a regular grid. These observables can be thought of as solutions to discrete analogues of special Riemann-type boundary value problems for holomorphic functions and an appropriate toolbox needed to prove their convergence was developed in [2].

Though spin correlations cannot be directly obtained as the values of discrete holomorphic functions, one can express their spatial derivatives via discrete holomorphic spinors that solve similar boundary value problems. When the convergence of these spinors is established, one can reconstruct the scaling limits of spin correlations from the asymptotic behaviour at singularities of the corresponding spinors in continuum [1]. Interestingly, one can use the same observables to give a short proof of some classical results about the diagonal spin-spin expectations in the full plane via orthogonal polynomials techniques.

This approach can be generalized to mixed correlations of fermions, spins, disorders and energy densities: all of them can be defined via solutions to a set of relevant boundary value problems, which paves the way to the proof of their convergence as the mesh size tends to zero. Moreover, one can recover asymptotics of these correlation functions at singularities from such a definition, thus rigorously establishing a standard set of CFT fusion rules for spins, disorders and fermions. The talk is mostly based on joint works with Clément Hongler (Lausanne) and Konstantin Izyurov (Helsinki).

## References

[1] Dmitry Chelkak, Clément Hongler, Konstantin Izyurov, Conformal invariance of spin correlations in the planar Ising model. *Ann. Math.* 181 (2015), no. 3, 1087–1138.

[2] Dmitry Chelkak, Stanislav Smirnov, Universality in the 2D Ising model and conformal invariance of fermionic observables, *Invent. Math,* 189 (2012), no. 3, 515–580.