

Derived functors in group theory and homotopy theory

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Every two-sided ideal \mathfrak{a} in the integral group ring $\mathbb{Z}[G]$ of a group G determines a normal subgroup $F \cap (1 + \mathfrak{a})$ of G , called a *generalized dimension subgroup*. Identification of such subgroups is a fundamental problem in the theory of group rings. It turns out that this problem leads to the theory of derived functors of non-additive functors, developed by Dold and Puppe in 60-s. From the other hand, the same derived functors as in the above problem, appear naturally in classical algebraic topology: in the theory of homotopy groups of Moore spaces and homology of Eilenberg-MacLane spaces. In the series of papers written jointly with S. Ivanov, I.B.S. Passi and J. Wu [1], [2], [3], the connection of derived functors and homotopy groups with group ring theory was developed. For example, homotopy groups of the 2-sphere are realized as dimension quotients for a certain choice of ideals in free group ring in [4]. Different examples of connections between derived functors and dimension subgroups will be given.

References

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- [4] R. Mikhailov, I.B.S. Passi and J. Wu: Symmetric ideals in group rings and simplicial homotopy, J. Pure Appl. Algebra 215 (2011), 1085-1092.