Martingale Optimal Transport

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In the classical optimal transport problem, one is given twoprobability measures μ, ν on $\mathbb{R}^d$ and a function c on $\Omega := \mathbb{R}^d \times \mathbb{R}^d$ and looksfor an extremal map that transportsμ to ν. The optimizationcriteria is given through c. The celebrated Kantarovich dualof this problem is to minimize $\mathcal{L}(h, g) := \int h d\mu + \int g d\nu$,over all integrable functions h, g of $\mathbb{R}^d$ that satisfy the inequality$h \oplus g(z) := h(x) + g(y) \geq c(z)$for every $z = (x, y) \in \Omega$.Then, the maximal value is given through by optimizing $\mathbb{E}_{\mathbb{Q}}[c]$over $\mathcal{M}(\mu, \nu)$, where a probability measures $\mathbb{Q}$ on Borel sets of Ω is in this setif it satisfies the marginalconstraints

$$\mathbb{E}_{\mathbb{Q}}[h \oplus \mathbf{0}] = \int h d\mu, \quad \mathbb{E}_{\mathbb{Q}}[\mathbf{0} \oplus g] = \int g d\nu, \quad \forall \, h, g,$$

where $\mathbf{0}$ is zero function and $\mathbb{E}_{\mathbb{Q}}$is the expectation with respect to the probability measure $\mathbb{Q}$.The *martingale optimal transport on* Ωreplaces the above constraint by requiring thatthere exists a measurable function γ such that

$$h \oplus g(z) + \gamma(x) \cdot (y - x) \geq c(z), \quad \forall z = (x, y) \in \Omega.$$

The dual of this problem is again to maximize$\mathbb{E}_{\mathbb{Q}}[c]$ but over a smaller set $\mathcal{Q}(\mu, \nu)$.Indeed, $\mathbb{Q} \in \mathcal{M}(\mu, \nu)$ is in this setif it is a martingale measure. Namely,

$$\mathbb{E}_{\mathbb{Q}}[\gamma(x) \cdot (y - x)] = 0$$

for every bounded measurable γ.Extensions of this problem to the cases when Ω is the setof continuous functions is given in [1] and to the Skorokhod space in [2]. In this talk, we outline these and related results in martingale optimal transport.

References

[1] DOLINSKY, Y. AND SONER, H. M. (2013).Martingale Optimal Transport and Robust Hedging inContinuous Time.*Probability Theory Related Fields.* **160(1-2)**, 391–427.

[2] DOLINSKY, Y. AND SONER, H. M. (2014).Martingale Optimal Transportin the Skorokhod Space. (2015),*Stochastic Processes and its Applications*,**125**, 3893-3931.