Cut Finite Element Methods

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Finite element methods are famous for efficiently solving Partial Differential Equations (PDEs) in complex geometries but require the mesh to conform to the geometry. When the geometry is evolving and undergoes strong deformations the required remeshing and interpolation [3] leads to significant complications, especially in three space dimensions.

We present a new computational method for solving PDEs in dynamic geometries. Such PDEs occur for example in multiphase flow problems where PDEs on interfaces separating immiscible fluids or in bulk domains having these interfaces as boundaries need to be solved. The proposed method, referred to as Cut Finite Element Methods (CutFEM), allows the dynamic geometry to be arbitrarily located with respect to a fixed background mesh. The strategy is essentially to embed the time-dependent domain where the PDE has to be solved in a fixed background mesh, equipped with a standard finite element space, and then take the restriction of the finite element functions to the time-dependent domain. Since the geometry can cut through the mesh arbitrarily there might be elements with small cuts. Such "small cut elements" may cause ill-conditioning and also prohibit the application of a whole set of well-known estimates, such as inverse inequalities. We add consistent stabilization terms [1] to the variational formulation which let us transfer the control of discrete functions on small cut elements to close-by neighbors with large intersection. These stabilization terms guarantee that the resulting system matrices have bounded condition number independently of the position of the dynamic geometry relative to the background mesh.

We have proposed stabilized CutFEM for the Stokes equations involving two immiscible incompressible fluids with different viscosities and with surface tension [4], for PDEs on time-dependent surfaces [5], for stationary coupled bulk-surface problems [2], and for time dependent coupled bulk-surface problems [6] modeling the evolution of soluble surfactants.

References


