Existence of knotted vortex structures in stationary solutions of the Euler equations

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The goal of this talk is to introduce recent results on the existence of knotted vortex lines and vortex tubes for stationary solutions to the Euler equations in $\mathbb{R}^3$. More precisely, given a finite collection of (possibly knotted and linked) pairwise disjoint closed curves and tubes, I will show that they can be transformed using a diffeomorphism of $\mathbb{R}^3$ into a set of vortex lines and vortex tubes of a Beltrami field that tends to zero at infinity [1, 2]. Beltrami fields are stationary solutions of the Euler equations with constant Bernoulli function, and also play an important role in magnetohydrodynamics (e.g. stellar atmospheres), where they are known as force-free magnetic fields. I will also discuss how to prove the existence of vortex lines and vortex tubes of arbitrarily complicated topology for high-energy Beltrami fields in the round sphere $\mathbb{S}^3$ and the flat torus $\mathbb{T}^3$ [3]. The problem of the existence of knotted vortex tubes in stationary inviscid fluids can be traced back to Lord Kelvin, and its experimental realization has only been achieved recently by Irvine and Kleckner at Chicago [4].

References

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