Congested transport at microscopic and macroscopic scales

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This contribution addresses the issue of congestion in the context of modeling the collective motion of active entities, in particular in the case of human crowds, at both microscopic and macroscopic levels. At the microscopic level, congestion takes the form of a non-overlapping constraint between entities. The dual formulation of this constraint leads to a pressure field defined on the contact points between individuals, and this pressure solves a degenerate discrete Laplace equation on the contact network ([1, 2]).

At the macroscopic level, the population is represented by a density field, that is subject to remain below a threshold value. Even in the simplest situation, namely congested transport, this setting raises delicate issues in terms of existence of solutions. We shall present how the Wasserstein setting (the distance between measures is defined by means of optimal transportation considerations, see [5, 6]) provides a natural framework for this kind of problems, and exhibits a continuous counterpart to the microscopic pressure as a Kantorovich potential associated to the underlying optimal transportation problem ([3, 4]).

We shall present the similarities, but also the deep discrepancies, between the microscopic and the macroscopic approaches. In particular, the discrete Laplace-like operator involved in the microscopic formulation lacks important properties, like the maximum principle, and this very feature makes it possible to recover non trivial phenomena, like the occurence of static jams, or the appearance of huge interaction forces between individuals within the crowd. In a more prospective approach, we shall address some questions raised by the inertial version of these models, namely granular flows (micro) and congested pressureless Euler equations (macro).

References

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